University of Wyoming

MATH 2200-02 – Fall 2026

Quiz 5 – Solutions

Problem 1 (2 points)**.** For the function $f(x) = x^3 - 6x^2$ determine

(a) The intervals on which f is increasing and decreasing.

(b) The local minima and maxima of f.

Answer 1. The function f is differentiable everywhere, and

$$f'(x) = 3x^2 - 12x = 3x(x-4).$$

Therefore, $f'(x) = 0$ if and only if $x = 0$ or $x = 4$, so these are the critical points of f. They divide the real line into three intervals, on each of which f' has a constant sign:

Interval	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
$3x$	$-$	$+$	$+$
$x-4$	$-$	$-$	$+$
f'	$+$	$-$	$+$
f	$\nearrow$	$\searrow$	$\nearrow$

(a) The function f is increasing on $(-\infty, 0)$ and on $(4, \infty)$, and it is decreasing on $(0, 4)$.

(b) We apply the first derivative test:

- At $x = 0$, the derivative f' changes from positive to negative, so f has a local maximum there. Its value is $f(0) = 0$.
- At $x = 4$, the derivative f' changes from negative to positive, so f has a local minimum there. Its value is $f(4) = 4^3 - 6 \cdot 4^2 = 64 - 96 = -32$.

Problem 2 (2 points)**.** Use L'Hôpital's rule to evaluate $\lim_{x\to 0} \frac{\sin x}{x}$.

Answer 2. As $x \to 0$, we have $\sin x \to 0$ and $x \to 0$, so the limit has the indeterminate form $\frac{0}{0}$ and L'Hôpital's rule applies. It gives

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} = \lim_{x\to 0} \frac{\cos x}{1} = \cos 0 = 1.$$