College Mathematics IV
§2.4 Probability of an Event

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Probability

- The likelihood of the occurrence of an event in a statistical experiment is evaluated by probabilities.
- Each sample point is assigned a probability or weight such that the sum of the all the weights is equal to 1.

Example (Flipping a Coin)

Assuming that each side occurs equally we assign the same weight $1/2 = 50\%$ to each side.

Example (Tossing a Die)

The sample space is $\{1, 2, 3, 4, 5, 6\}$.

We assign to each number the same weight $1/6 = 16.67\%$.

Probability of an Event

Definition

The probability $P(A)$ of an event A is equal to the sum of the weights of all sample points in A.

Example (Flipping a Coin)

The probability to get a head is $1/2 = 50\%$.

Example (Tossing a Die)

The sample space is $\{1, 2, 3, 4, 5, 6\}$.

- The probability to get a 6 is $1/6 = 16.67\%$.
- The probability to get an even number is equal to

$$(\text{weight of } 2) + (\text{weight of } 4) + (\text{weight of } 6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}.$$

Properties of Probalities

Properties

1. *For every event A, we have*
$$0 \leq P(A) \leq 1.$$
2. *We always have*
$$P(\emptyset) = 0 \qquad \text{and} \qquad P(S) = 1.$$
3. *If $A_1, A_2, \ldots$ are mutually disjoint events, then*
$$P\big(A_1 \cup A_2 \cup \cdots\big) = P(A_1) + P(A_2) + \cdots .$$

Properties of Probalities

Example

Let A be an event and A' its complement in S.

- As $A \cap A' = \emptyset$ and $A \cup A' = S$, we have

$$1 = P(S) = P\left(A \cup A'\right) = P(A) + P(A').$$

- Thus,

$$P(A') = 1 - P(A).$$

Properties of Probalities

Example (Tossing a Die)

A die is loaded in such a way that an even number is twice as likely to occur as an odd number.

1. If E is the event that a number less than 4 occurs on a single toss of the die, find $P(E)$.
2. Let A be the event that an even number turns up and B the event that a number divisible by 3 occurs. Find $P(A \cup B)$ and $P(A \cap B)$.

Properties of Probalities

Rule

Let S be a sample space with N elements that are equally likely to occur.

1. Each sample point has probability/weight $1/N$.
2. If A is an event containing n sample points, then

$$P(A) = \frac{n}{N}$$

Properties of Probalities

Example

A statistics class for engineers consists of 15 civil, 10 electrical, 15 software engineering students. If a student is randomly asked a question by the instructor, then find the probability for that student to be

(a) An electrical engineering student.

(b) A civil or software engineering student.