

College Mathematics II
§2.2 The Derivative as a Function

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The Derivative of a Function

Reminder

The derivative at x of a function f is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

Definition

The **derivative** of f is the function that assigns to x the number $f'(x)$ wherever it is defined.

The Derivative of a Function

Example

If $f(x) = x^3 - x$, then find f' .

Remark (Alternative Notations)

- There are various notations for the derivative

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f = Df(x) = D_x f(x).$$

- The notation $\frac{dy}{dx}$ is called **Leibniz notation**.

Definition

- A function f is **differentiable** at a if $f'(a)$ exists.
- It is differentiable on an open interval (a, b) (or $(-\infty, b)$, or (a, ∞) , or $(-\infty, \infty)$) if it is differentiable at every point of the interval.

Example

Where is the function $f(x) = |x|$ differentiable?

Theorem

If f is differentiable at a , then f is continuous at a .

Remark

- The converse does not hold.
- The function $f(x) = |x|$ is continuous at 0 , but it is not differentiable there.

Higher Order Derivatives

Definition

The derivative of f' is denoted f'' and is called the **second derivative** of f .

Example

If $f(x) = x^3 - x$, then find f'' .

Remark

Alternative notations for the second derivative:

$$y'' = f'' = f^{(2)} = \frac{d^2}{dx^2} f = \frac{d^2 f}{dx^2}.$$

Definition

The **third derivative** of f is the derivative of f'' and is denoted f''' .

Remark

Alternative notations for the third derivative:

$$y''' = f''' = f^{(3)} = \frac{d^3}{dx^3} f = \frac{d^3 f}{dx^3}.$$

Higher Order Derivatives

Remark

- We similarly define the **fourth derivative** $f^{(4)}$, the **fifth derivative** $f^{(5)}$, etc..
- The n -th derivative is denoted $f^{(n)}$ or $\frac{d^n f}{dx^n}$.

Example

If $f(x) = x^3 - x$, then find f''' and $f^{(4)}$.